Conditional Cube Attacks on KECCAK-p Based Constructions

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Outlines

1. KECCAK
2. Conditional Cube Attacks
3. New MILP Model for Searching Conditional Cubes
4. Main Results

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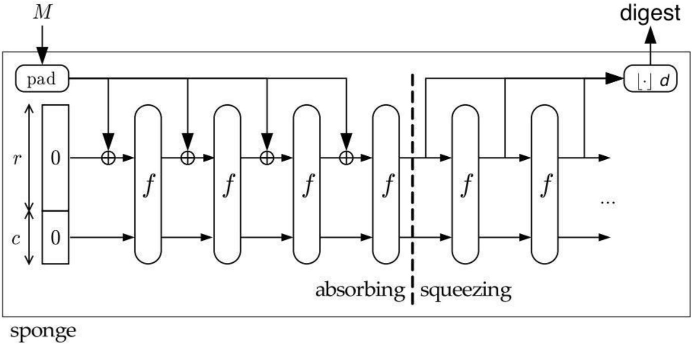
1. KECCAK
2. Conditional Cube Attacks
3. New MILP Model for Searching Conditional Cubes
4. Main Results

SHA-3 (KECCAK) Hash Function

The sponge construction [BDPV11]



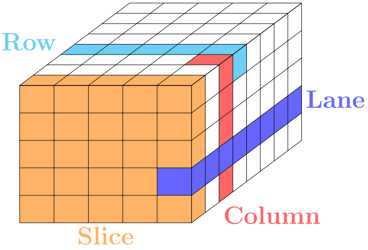
- b-bit permutation f
- Two parameters: bitrate r, capacity c, and $b = r + c$.
- The message is padded and then split into r-bit blocks.

KECCAK Permutation

- 1600 bits: seen as a 5×5 array of 64-bit lanes, $A[x, y], 0 \leq x, y < 5$
- 24 rounds
- each round R consists of five steps:

$$R = \iota \circ \chi \circ \pi \circ \rho \circ \theta$$

- χ : the only nonlinear operation



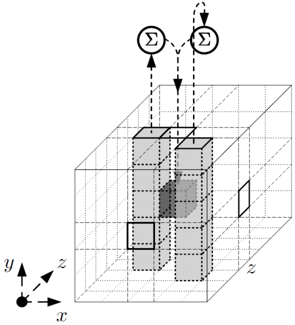
http://www.iacr.org/authors/tikz/

Keccak Permutation

Round function: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

θ step: adding two columns to the current bit

$$
\begin{aligned}
C[x] =& A[x,0] \oplus A[x,1] \oplus A[x,2] \oplus \\
& A[x,3] \oplus A[x,4] \\
D[x] =& C[x-1] \oplus (C[x+1] \lll 1) \\
A[x,y] =& A[x,y] \oplus D[x]
\end{aligned}
$$



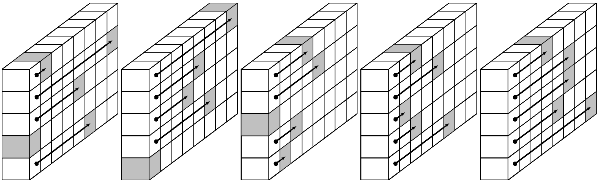
http://keccak.noekeon.org/

- The Column Parity kernel
  - If $C[x] = 0, 0 \leq x < 5$, then the state A is in the CP kernel.

KECCAK Permutation

Round function: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

ρ step: lane level rotations, $A[x, y] = A[x, y] \lll r[x, y]$



http://keccak.noekeon.org/

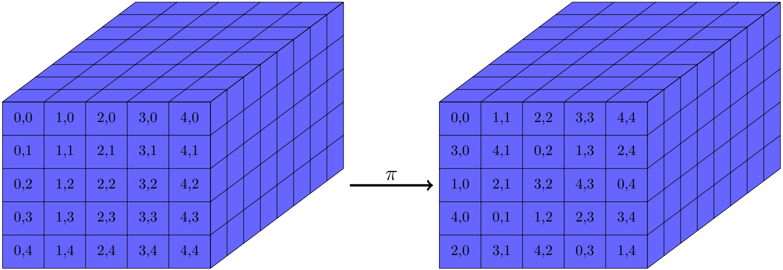
Rotation offsets $r[x, y]$

	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$y = 0$	0	1	62	28	27
$y = 1$	36	44	6	55	20
$y = 2$	3	10	43	25	39
$y = 3$	41	45	15	21	8
$y = 4$	18	2	61	56	14

Keccak Permutation

Round function: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

π step: permutation on lanes



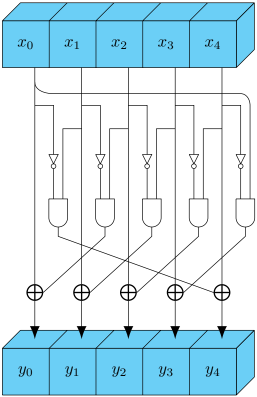
$$A[y, 2*x+3*y] = A[x, y]$$

KECCAK Permutation

Round function: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

χ step: 5-bit S-boxes, nonlinear operation on rows

$$y_0 = x_0 + (x_1 + 1) \cdot x_2,$$
$$y_1 = x_1 + (x_2 + 1) \cdot x_3,$$
$$y_2 = x_2 + (x_3 + 1) \cdot x_4,$$
$$y_3 = x_3 + (x_4 + 1) \cdot x_0,$$
$$y_4 = x_4 + (x_0 + 1) \cdot x_1.$$

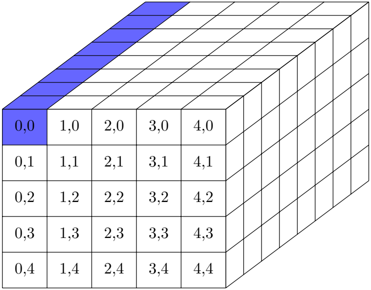


Keccak Permutation

Round function: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

ι step: adding a round constant to the state

Adding one round-dependent constant to the first "lane", to destroy the symmetry.



$$A[0,0] = A[0,0] \oplus RC[i]$$

KECCAK Permutation

Round function

Internal state A: a 5×5 array of 64-bit lanes

θ step $C[x] = A[x, 0] \oplus A[x, 1] \oplus A[x, 2] \oplus A[x, 3] \oplus A[x, 4]$
$D[x] = C[x - 1] \oplus (C[x + 1] \lll 1)$
$A[x, y] = A[x, y] \oplus D[x]$

ρ step $A[x, y] = A[x, y] \lll r[x, y]$
- The constants $r[x, y]$ are the rotation offsets.

π step $A[y, 2 * x + 3 * y] = A[x, y]$

χ step $A[x, y] = A[x, y] \oplus ((\ A[x + 1, y])\&A[x + 2, y])$

ι step $A[0, 0] = A[0, 0] \oplus RC[i]$
- $RC[i]$ are the round constants.

The only non-linear operation is χ step.

KECCAK-p Based Constructions

KMAC

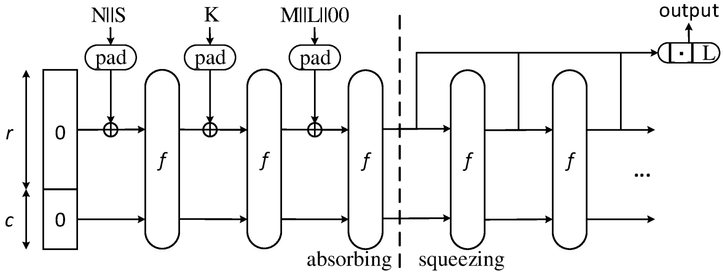
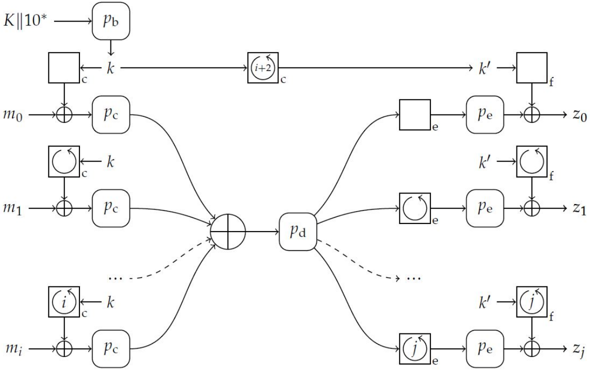


Figure: KMAC processing one message block

- Two versions: KMAC128 and KMAC256
- N and S are public strings.

KECCAK-p Based Constructions

KRAVATTE



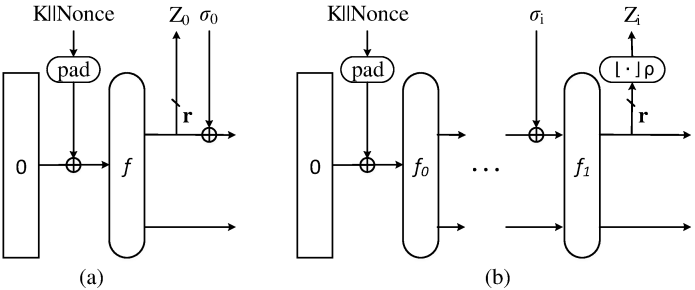
▢ stands for permutations and □ symbolizes rolling functions.

$$p_b = p_c = \text{KECCAK-}p[1600, 6],$$
$$p_d = p_e = \text{KECCAK-}p[1600, 4]^1.$$

[1]Version of 17-Jul-2017.

KECCAK-p Based Constructions

KEYAK and KETJE



(a) KEYAK and (b) KETJE

Outline

Cube Attacks [DS09]

- Given a Boolean polynomial $f(k_0, \ldots, k_{n-1}, v_0, \ldots, v_{m-1})$ and a monomial $t_I = \wedge_{i_r \in I} v_{i_r}$, $I = (i_1, \ldots, i_d)$, f can be written as

$$f(k_0, \ldots, k_{n-1}, v_0, \ldots, v_{m-1}) = t_I \cdot p_{S_I} + q(k_0, \ldots, k_{n-1}, v_0, \ldots, v_{m-1})$$

  - q contains terms that are not divisible by t_I
  - p_{S_I} is called the superpoly of I in f
  - $v_{i_1}, \ldots, v_{i_d}$ are called cube variables. d is the dimension.
- The the cube sum is exactly

$$p_{S_I} = \sum_{(v_{i_1}, \ldots, v_{i_d}) \in C_I} f(k_0, \ldots, k_{n-1}, v_0, \ldots, v_{m-1})$$

- Cube attacks: p_{S_I} is a low-degree polynomial in key bits.
- Cube testers: distinguish p_{S_I} from a random function. E.g., $p_{S_I} = 0$.

Conditional Cube Testers of KECCAK [HWX+17]

- Ordinary cube variables:
  - Do not multiply with any variable in the **first** round.
- Conditional cube variables:
  - Do not multiply with any variable in the **first two** rounds under certain conditions.

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Properties

- 2^n-dimensional cubes with 1 conditional cube variable
  - The cube sum is **zero** for $(n+1)$-round KECCAK.

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Properties

- 2^n-dimensional cubes with 1 conditional cube variable
  - The cube sum is **zero** for $(n+1)$-round KECCAK.
- If the conditions involve the key, the conditional cube can be used to recover the key.
- Time complexity of the key recovery: $\mathbf{\frac{k}{t} \cdot 2^{2^n+t}}$, where t is the number of key bits involved in the conditions.

Outline

How to keep the first χ linear

The expression of $b = \chi(a)$ is of algebraic degree 2:

$b_i = a_i + \overline{a_{i+1}} \cdot a_{i+2}$, for $i = 0, 1, \ldots, 4$.

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Observation

When there is no neighbouring variables in the input of an Sbox, then the application of χ does NOT increase algebraic degree.

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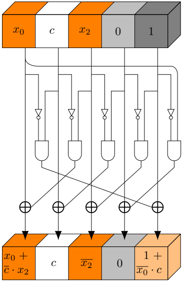

√

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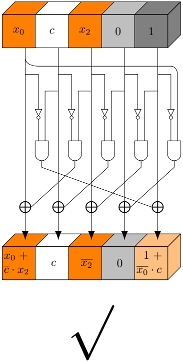
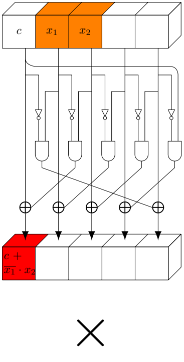




Linear Structure [GLS16]

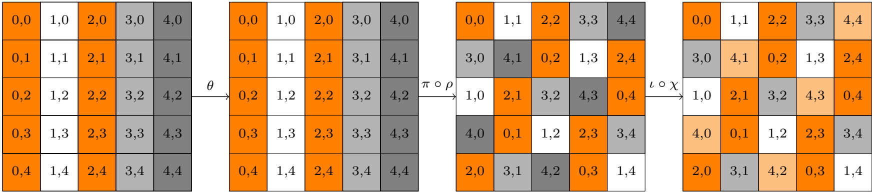


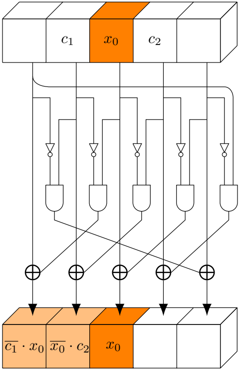
Figure: 1-round linear structure of KECCAK-p with the degrees of freedom up to 512, where ■: variables; ■: algebraic degree at most 1; ■: 1; ■: 0.

- All variables do not multiply with each other in the first round.
- **BUT** we need at least one conditional variable.

The Conditional Cube variable

Requirement of the second χ

- If an input bit of the **second** χ contains the conditional variable, then its neighbouring bits should be constants.
- These neighbouring bits are denoted as $s_0, s_1, \ldots$
- Each s_i is calculated from 11 output bits of the first round.



New MILP Model

Mixed integer linear programming (MILP) takes an *objective function* *obj* and a set of inequalities $M \cdot X < b$ over real numbers as input and finds solutions optimizing *obj*.

Let $a[x][y][z]$ be the state:

$$a \xrightarrow{\pi \circ \rho \circ \theta} b \xrightarrow{\chi} c$$

$A[x][y][z] = 1$ if $a[x][y][z]$ contains a cube variable:

$$A \xrightarrow{\pi \circ \rho \circ \theta} B \xrightarrow{\chi} C$$

$V[x][y][z] = 1$ indicates a bit condition.

Modeling the First χ

Patterns of the Diffusion of χ

$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]^{1}$$

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$

[1] Omit coordinates $[y][z]$.

Modeling the First χ

Patterns of the Diffusion of χ

$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]^1$$

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$
constant	constant	constant	constant

[1]Omit coordinates $[y][z]$.

Modeling the First χ

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$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]^1$$

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$
constant	constant	constant	constant
var	*	*	var

[1]Omit coordinates $[y][z]$.

Modeling the First χ

Patterns of the Diffusion of χ

$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]^1$$

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$
constant	constant	constant	constant
var	*	*	var
constant	constant	var	var (deg ≤ 1)

[1] Omit coordinates $[y][z]$.

Modeling the First χ

Patterns of the Diffusion of χ

$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]^1$$

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$
constant	constant	constant	constant
var	*	*	var
constant	constant	var	var (deg ≤ 1)
constant	1	var	constant

[1]Omit coordinates $[y][z]$.

Modeling the First χ

Patterns of the Diffusion of χ

$$c[x] = b[x] + \overline{b[x+1]} \cdot b[x+2]$$ [1]

$b[x]$	$b[x+1]$	$b[x+2]$	$c[x]$
constant	constant	constant	constant
var	*	*	var
constant	constant	var	var $(\deg \leq 1)$
constant	1	var	constant
⋮	⋮	⋮	⋮

[1] Omit coordinates $[y][z]$.

Modeling the First χ

Patterns of the Diffusion of χ

$$B[x] = \begin{cases} 0, & b[x] \text{ is a constant;} \\ 1, & b[x] \text{ is a var.} \end{cases} \qquad V[x] = \begin{cases} 0, & \text{no condidtion on } b[x]; \\ 1, & b[x] \text{ is restricted to } 0/1. \end{cases}$$

Table: Diffusion of variables through χ. Symbol '*' denotes arbitrary value.

$B[x]$	$B[x+1]$	$B[x+2]$	$V[x+1]$	$V[x+2]$	$C[x]$
0	0	0	*	*	0
1	0	0	*	*	1
1	0	1	*	0	1
0	0	1	0	0	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	0	1	0

Modeling the First χ

Inequalities Describing the Diffusion of χ

- By generating the convex hull of the set of patterns, we get

$$
\begin{aligned}
B[x] - B[x+1] - B[x+2] - V[x+1] - V[x+2] - C[x] &\geq -2 \\
-B[x] - B[x+1] + V[x+2] + C[x] &\geq 0 \\
-B[x+2] - V[x+2] &\geq -1 \\
B[x] + B[x+1] + B[x+2] - C[x] &\geq 0 \\
-B[x] + C[x] &\geq 0 \\
-B[x+1] - B[x+2] + V[x+1] + V[x+2] + C[x] &\geq 0 \\
-B[x] - B[x+1] &\geq -1
\end{aligned}
$$

Modeling the Second χ

Two Cases for the Second χ

- Each neighbouring bit s_i of the conditional variables is calculated from 11 bits of $c[x][y][z]$.

  Case 1 For these 11 bits, none of them are variables, i.e., $C[x][y][z] = 0$;

  Case 2 There are variables among the 11 bits and the XOR of these bits forms a linear equation which consumes 1 bit degree of freedom.

- Introduce S_i for s_i

$$S_i = \begin{cases} 0, & \text{for Case 1;} \\ 1, & \text{for Case 2.} \end{cases}$$

Modeling the Second χ

Patterns and Inequalities for the Second χ

If $c[x][y][z]$ is needed for calculating s_i, then $c[x][y][z]$ should not contain terms with uncertain coefficients.

- Patterns that exclude terms with uncertain coefficients:

S_i	$B[x]$	$B[x+1]$	$B[x+2]$	$V[x+1]$	$V[x+2]$
0	*	*	*	*	*
1	0	0	0	*	*
1	1	0	0	*	*
1	1	0	1	**1**	0
1	0	0	1	1	0
1	0	1	0	0	1

Modeling the Second χ

Patterns and Inequalities for the Second χ

- Inequalities:

$$\begin{aligned}
-S_i - B[x+1] - B[x+2] &\geq -2 \\
-S_i + B[x] - B[x+1] + V[x+2] &\geq -1 \\
-S_i - B[x+2] + V[x+1] &\geq -1 \\
-S_i - B[x+1] - V[x+1] &\geq -2 \\
-S_i - B[x+2] - V[x+2] &\geq -2 \\
-S_i - B[x] - B[x+1] &\geq -2
\end{aligned}$$

Modeling the Search for Conditional Cubes

- Modeling the linear layer is simple.
- Set the dimension of the target cube to 2^n.
- Objective

$$\text{Minimize}: \quad \sum V[x][y][z].$$

Outline

Application of the New Model

The new model is applicable to keyed KECCAK modes, including

- Constructions with fully unknown internal state
  - KMAC, KRAVATTE (**first attacks**)
- Constructions with partially known internal state
  - KETJE, KEYAK (**improved attacks**)

KMAC and KRAVATTE

Target	Key Size	Capacity	n_r Rounds	Complexity	Reference
KMAC128	128	256	7	2^{76}	this
KMAC256	256	512	9	2^{147}	
KRAVATTE	128	-	8	2^{65}	this
	256	-	9	2^{129}	
KECCAK-MAC	128	256/512	7	2^{72}	[HWX+17]
		768	7	2^{75}	[LBW+17]
		1024	6	$\mathbf{2^{58.3}}$	
		1024	6	$\mathbf{2^{41}}$	this

KEYAK and KETJE

Target	Key Size	n_r Rounds	Complexity	Nonce respected	Reference
Lake KEYAK	128	**6**	2^{37}	Yes	[DMP+15]
	128	8	2^{74}	No	[HWX+17]
	128	**8**	$2^{71.01}$	Yes	this
	256	9	$2^{137.05}$	Yes	
River KEYAK	128	8	2^{77}	Yes	
KETJE Major	128	7	2^{83}	Yes	[LBW+17]
	128	7	$\mathbf{2^{71.24}}$	Yes	this
KETJE Minor	128	7	2^{81}	Yes	[LBW+17]
	128	7	$\mathbf{2^{73.03}}$	Yes	this
KETJE SR v1	128	7	2^{115}	Yes	[DLWQ17]
	128	7	$\mathbf{2^{92}}$	Yes	this

In conclusion:

1. Model the non-linear layer completely, and nest the two nonlinear layers in two rounds together.
2. First attacks on KMAC and KRAVATTE, and improved attacks on KEYAK and KETJE.

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Thank you for your attention!